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Галстян

Галстян Арсен Хачатурович

**ПРОБЛЕМА ФЕРМА–ШТЕЙНЕРА В
ГИПЕРПРОСТРАНСТВАХ**

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E-mail: vladimir.manuilov@gmail.com

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Общая характеристика работы

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:

(Y, ρ)

$A = \{A_1, \dots, A_n\}$

(_____)

¹A. Ivanov A., Tropin A., Tuzhilin A. Fermat–Steiner problem in the metric space of compact sets endowed with Hausdorff distance // J. Geom., 108:2 (2017), 575–590.

(Y, ρ) , ...).

2 3 4 5

7

6

(n)

(²)

y

A

y

A

²Ivanov A. O., Tuzhilin A. A. Branching solutions to one-dimensional variational problems // World Sci. Publ., River Edge, NJ, 2001, xxii+342 pp.

³Cieslik D. Steiner minimal trees // Nonconvex Optim. Appl., 23, Kluwer Acad. Publ., Dordrecht, 1998, xii+319 pp.

⁴Ivanov A. O., Tuzhilin A. A. Minimal networks: a review // Advances in dynamical systems and control, Stud. Syst. Decis. Control, 69, Springer, Cham, 2016, 43–80 pp.

⁵Hwang F. K., Richards D. S., Winter P. The Steiner Tree Problem // North-Holland, 1992, 339 p.

⁶Jarnik V., Kössler M. On minimal graphs containing n given points // Časopis Pěst. Mat. Fys., 63:8 (1934), 223–235 pp.

⁷... // ... 2001, 568 ...

$$A = \{A_1, \dots, A_n\}$$

$$y \in Y,$$

$$(Y, \rho)$$

$$S(A, y) = \sum_i \rho(y, A_i)$$

A

$\Sigma(A)$

A

A

A

8

k

3

$k = 1$

$\Sigma(A)$

()

A , 1

9 10

1.1.1.

A

X .

$p \in X$

A

$$|p A| = \inf\{|pa| : a \in A\}.$$

⁸Drezner Z., Klamroth K., Schöbel A., Wesolowsky G.O. The Weber problem // Facility Location: Applications and Theory, Springer, Berlin, 2002, 1–36 pp.

⁹Schlicker S. The geometry of the Hausdorff metric // GVSU REU 2008, Grand Valley State Univ., Allendale, MI, 2008, 11 pp., <http://faculty.gvsu.edu/schlicks/HMG2008.pdf>.

¹⁰

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// - :

$$A = \emptyset,$$

$$|p\emptyset| = \infty.$$

1.2.1. A

$$B_r(A) = \{p : |pA| \leq r\}; \quad U_r(A) = \{p : |pA| < r\}$$

A r .

1.4.2.

B

A

$$d_H(A, B) = \inf\{r : A \subset B_r(B), B \subset B_r(A)\}.$$

$$Y = \mathcal{H}(X)$$

$X,$

$\mathcal{H}(X)$

$$Y = \mathcal{H}(X)$$

¹¹

$X.$

$\mathcal{H}(X)$

$$\mathcal{H}(X), \quad \text{12, 13 14 15}$$

$\mathcal{H}(X)$

$$\mathcal{H}(X)$$

$$x \in X,$$

$A_i,$

¹¹Nadler S. B. Hyperspaces of sets // Marcel Dekker Inc., New York and Basel, 1978, 707 p.

¹²Blackburn C. C., Lund K., Schlicker S., Sigmon P., Zupan A. An introduction to the geometry of $\mathcal{H}(\mathbb{R}^n)$ // GVSU REU 2007, Grand Valley State Univ., Allendale, MI, 2007.

¹³Memoli F. On the use of Gromov–Hausdorff distances for shape comparison // Eurographics symposium on point based graphics, The Eurographics Association, Prague, 2007, 81–90.

¹⁴Memoli F. Some properties of Gromov–Hausdorff distances // Discrete Comput. Geom., 48:2 (2012), 416–440.

¹⁵Ivanov A. O., Tuzhilin A. A. Isometry group of Gromov–Hausdorff space // Mat. Vesnik, 71:1-2 (2019), 123–154.

$(A_i, \dots, A_n) \in \mathcal{H}(X)$
 X
 $K \in \Sigma(A)$
 $A_i, A_j \in K$
 X

$$A \subset \mathcal{H}(X), \quad \Sigma(A)$$

$$K \in \Sigma(A) : d(K, A) = (d_H(K, A_1), \dots, d_H(K, A_n))$$

$$\Omega(A) = \{d(K, A) : K \in \Sigma(A)\}$$

$$\Sigma_d(A) = \{K \in \Sigma(A) : d(K, A) = d\}$$

$$\Sigma_d(A), d \in \Omega(A), A_i \in A$$

$$\Sigma_d(A)$$

$$\Sigma_d(A)$$

$$\Sigma_d(A)$$

$$K \subset X$$

$$K$$

$$\Sigma_d(A)$$

$$A = \{A_1, A_2, A_3\} \subset \mathcal{H}(\mathbb{R}^2)$$

$$A_i, A_j$$

$\pm 2\pi/3$, O , 2.16, 1
 $\Sigma_d(A)$, 4-
 O , $\pm 2\pi/3$, 3,
 2.18, $S(A, K)$, 3, $(K = O)$
 $\mathcal{H}(X)$, X , \mathbb{R} , $\mathcal{H}(X)$,
 $K \in \mathcal{H}(X)$,
 $S(A, K) = d_H(A_1, K) + \dots + d_H(A_n, K)$.
 $S(A, K)$, S_A , $\Sigma(A)$,
 1, $\Sigma(A)$
 $K \in \Sigma(A)$.
 K , $A_i \in A$, d_i , $d = (d_1, \dots, d_n)$, A
 $\Omega(A)$, $\Omega(A)$, $\Sigma(A)$
 d , $\mathcal{H}(X)$
 $\Sigma_d(A)$,
 $d \in \Omega(A)$, 1
 $\Sigma_d(A)$

$$K_d = \bigcap_{i=1}^n B_{d_i}(A_i), \quad B_{d_i}(A_i) \quad (K_d) \quad , \quad d \in \Omega(A),$$

$$K_\lambda \in \Sigma_d(A)$$

$$K_\lambda \subset K \subset K_d.$$

2.0.1. $A,$

$$2.4.1. \quad A = \{A_1, \dots, A_n\} \subset \mathcal{H}(X), \quad \mathcal{H}(X)$$

$$X \quad \text{Conv}(K) \quad K.$$

$$\{ \text{Conv}(A_1), \dots, \text{Conv}(A_n) \} \quad A^{\text{Conv}}.$$

$$A \quad S_A = S_{A^{\text{Conv}}},$$

$$2.2.1. \quad a \quad A_i \in A$$

$$\tilde{d} = (\tilde{d}_1, \dots, \tilde{d}_n) \in \mathbb{R}^n, \quad \tilde{d}_j \geq 0,$$

$$U_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) = \emptyset.$$

$$A_i \in A \quad F_{\tilde{d}}^{A_i} \quad F_{\tilde{d}}^A = \bigcup_j F_{\tilde{d}}^{A_j}.$$

$$2.2.2. \quad a \quad A_i \in A$$

$$\tilde{d} = (\tilde{d}_1, \dots, \tilde{d}_n) \in \mathbb{R}^n,$$

$$\tilde{d}_j \geq 0, \quad \text{Int}(B_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j)) = \emptyset.$$

$$\tilde{d} \quad A_i \in A \quad L_{\tilde{d}}^{A_i}.$$

$$L_{\tilde{d}}^A = \bigcup_j L_{\tilde{d}}^{A_j}.$$

$$\begin{array}{l}
2.1.2. \quad a \quad A_i \in A \quad \tilde{d} = (\tilde{d}_1, \dots, \tilde{d}_n) \in \mathbb{R}^n, \quad \tilde{d}_j \geq 0, \\
\#B_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) < \infty. \quad \tilde{d} \\
A_i \in A \quad D_{\tilde{d}}^{A_i}. \quad D_{\tilde{d}}^A = \bigcup_j D_{\tilde{d}}^{A_j}. \\
\tilde{d} \in \mathbb{R}^n, \quad \tilde{d}_j \geq 0 \\
j. \quad Y_{\tilde{d}}^{A_i} \in \{F_{\tilde{d}}^{A_i}, L_{\tilde{d}}^{A_i}, D_{\tilde{d}}^{A_i}\} \quad Y_{\tilde{d}}^A \in \{F_{\tilde{d}}^A, L_{\tilde{d}}^A, D_{\tilde{d}}^A\}. \quad F_{\tilde{d}}^{A_i}, L_{\tilde{d}}^{A_i}, D_{\tilde{d}}^{A_i}, \\
- \text{HP}(p, Y_{\tilde{d}}^{A_i}) := B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j), \quad p \in Y_{\tilde{d}}^{A_i}; \\
- \text{HP}(Y_{\tilde{d}}^{A_i}) := \bigcup_{p \in Y_{\tilde{d}}^{A_i}} \text{HP}(p, Y_{\tilde{d}}^{A_i}); \\
- \text{HP}(Y_{\tilde{d}}^A) := \bigcup_i \text{HP}(Y_{\tilde{d}}^{A_i}). \\
\tilde{d} = d \in \Omega(A), \quad \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) = K_d \\
\Sigma_d(A). \quad \text{HP}(p, Y_d^{A_i}) = \\
B_{d_i}(p) \cap \bigcap_{j=1}^n B_{d_j}(A_j) = B_{d_i}(p) \cap K_d, \quad p \in Y_d^{A_i}. \\
\text{HP}(p, Y_{\tilde{d}}^{A_i}) \quad Y_{\tilde{d}}^{A_i} \\
B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j). \\
p \in F_{\tilde{d}}^{A_i} \quad U_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) = \emptyset; \quad p \in L_{\tilde{d}}^{A_i} \quad \text{Int}(B_{\tilde{d}_i}(p) \cap \\
\bigcap_{j=1}^n B_{\tilde{d}_j}(A_j)) = \emptyset; \quad p \in D_{\tilde{d}}^{A_i} \quad \#B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) < \infty. \\
2.4.2. \quad K_\lambda \in \Sigma_d(A) \\
K_\lambda \setminus \text{HP}(D_d^A) \subset \text{Int } K_d. \\
X \quad \mathbb{R}. \\
" \quad " \\
1.
\end{array}$$

– 2 1 1 (")

$R(K)$

$K \subset X$

$(p, a) \in R(K), \quad p \in K \quad a \in A_i, \quad |pa| \leq d_i.$
 1
 $P \quad Q, \quad Q$

$C_i.$

$p \in P$

$C_i \quad p$

$q \in Q, \quad q$

$p, \quad 1.$

A

A

K

$R(K)$

(

),

1,

K

$\Sigma_d(A).$

K

– 2 1 4 (")

A

K_λ

\tilde{A}

n

$A.$

$$\#K_\lambda \leq \#\tilde{A} - n + 1,$$

A_i

$$\#K_\lambda \leq \#\tilde{A} - n.$$

X

$$\#K_\lambda \leq \#\tilde{A} - n.$$

2.

2

4

1 2 (

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)

$$A = \{A_1, \dots, A_n\}$$

$$A^{\text{Conv}} = \{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\}.$$

d

A .

—

$$A = \{A_1, \dots, A_n\}$$

$$d \in \Omega(A).$$

$$A^{\text{Conv}} = \{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\}.$$

i

$$F_d^{\text{Conv}(A_i)} = \emptyset$$

i

$$L_d^{\text{Conv}(A_i)} = \emptyset,$$

A

—

$$A = \{A_1, \dots, A_n\}$$

d_i

$$d \in \Omega(A).$$

$$A^{\text{Conv}} = \{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\}.$$

s

$$\text{HP}(F_d^{\text{Conv}(A_s)}) = \emptyset$$

$$p \in \text{HP}(F_d^{A^{\text{Conv}}}) \quad p \notin \partial B_{d_s}(\text{Conv}(A_s)),$$

A

—

$$(1) \quad X \quad ;$$

$$(2) \quad A = \{A_1, \dots, A_n\} \quad ;$$

$$(3) \quad U_d^{\text{Conv}} = \text{Int } K_d^{\text{Conv}} \neq \emptyset, \quad d \in \Omega(A);$$

$$(4) \quad d_s > 0;$$

$$(5) \quad \left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \text{HP}(D_d^A) \subset U_d^{\text{Conv}}, \quad m_s$$

$A_s.$

(

$$\text{HP}(D_d^A) = \text{HP}(L_d^A), \quad (1) \quad (2) \quad (5)$$

$$\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \text{HP}(L_d^A) \subset$$

$U_d^{\text{Conv}}.$

$$(1) \quad (2) \quad \text{Cl}(\text{Int } K_d) = K_d,$$

$$\text{HP}(D_d^A) = \text{HP}(L_d^A) = \text{HP}(F_d^A), \quad (5)$$

$$\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap$$

$$\text{HP}(F_d^A) \subset U_d^{\text{Conv}}.)$$

$$(1)-(5) \quad A.$$

$$(1)-(5)$$

$$K_\lambda \in \Sigma_d(A)$$

$$\delta_1 := \left| K_\lambda \partial B_{d_s}(\text{Conv}(A_s)) \right| > 0;$$

$$\delta_2 := \left| \text{Conv}(A_s) \partial B_{d_s}(K_d^{\text{Conv}}) \right| > 0.$$

$$\min\{\delta_1, \delta_2, d_s\} > 0.$$

$$0 < \delta \leq$$

$$\min\{\delta_1, \delta_2, d_s\} \quad K = B_{d_s-\delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}}.$$

:

$$\begin{aligned} S_A - S_{A^{\text{Conv}}} &\geq S_A - S(A^{\text{Conv}}, K) \geq \\ &\geq S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S(A^{\text{Conv}}, K) \geq \delta > 0. \end{aligned}$$

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2.1.7 2.4.3

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	1.6	1			

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 - XVII - " -2018",
23-28 2018;
 - XXVI " -2019",
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27 -4 2020;

– XXIX
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1.6

1.6

2.1

$\Sigma_d(A)$

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2.1.1

$K \in \mathcal{H}(X)$

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2.2),

2.1.2

$d(1)$

2.1.3

1.5 1

2.1.4

(2.6 2.7).

2.1.5

$$D_{\tilde{d}}^{A_i} \quad A_i \quad \tilde{d} = (\tilde{d}_1, \dots, \tilde{d}_n) \in \mathbb{R}^n, \quad \tilde{d}_i$$

HP($p, D_{\tilde{d}}^{A_i}$)

$$\bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) \quad p \in D_{\tilde{d}}^{A_i},$$

$$B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j).$$

X

d

$D_d^{A_i}$

$$\bigcup_{p \in D_d^{A_i}} \text{HP}(p, D_d^{A_i})$$

A_i

2.4

4

2.1.5

2.1

1

2.8

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$\Sigma_d(A)$.

X

2.1.5

i

K_λ

d_i

d_k

6 (

$p \in X$

d_i ,

$a \in A_i$

$|ap| = d_i$).

2.1.6

2.12

2.1.7

1

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2.18

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2.2

:

$$L_{\tilde{d}}^{A_i} \in \mathbb{R}^n$$

A_i

A_i

$\tilde{d} \in \mathbb{R}^n$

\tilde{d} ,

$$\text{HP}(p, Y_{\tilde{d}}^{A_i}) \quad \left(\bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) \quad p \in Y_{\tilde{d}}^{A_i}, \quad Y \in \{D, F, L\}, \right. \\ \left. B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j). \right. \\ \left. \text{2.21}, \right)$$

$$\left(\begin{array}{l} D_d^{A_i}, F_d^{A_i}, L_d^{A_i} \\ X \end{array} \right), \quad \text{2.23}, \quad \left(\begin{array}{l} A_i \\ F_d^{A_i} \end{array} \right), \\ \text{Cl}(\text{Int } K_d) = K_d, \quad K_d \\ \Sigma_d(A).$$

$$\text{2.28} \quad \left(\begin{array}{l} K_d \\ \text{2.28} \end{array} \right), \quad \left(\begin{array}{l} A_i \\ \partial B_{d_i}(A_i) \\ A_j. \end{array} \right) \\ \bigcup_{p \in F_d^{A_j}} \text{HP}(p, F_d^{A_j})$$

$$\text{2.28} \quad \left(\begin{array}{l} \text{2.4} \\ \text{12).} \end{array} \right), \quad \left(\begin{array}{l} A_i \\ \text{Conv}(A_i). \end{array} \right)$$

$$S(\{A_1, \dots, A_n\}, K) \quad \left(\begin{array}{l} A = \{A_1, \dots, A_n\}, \\ S(\{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\}, K) \end{array} \right)$$

$$\text{2.4.1}, \quad \left(\begin{array}{l} \text{2.4} \\ \text{2.4.2} \\ \text{2.4.1}, \end{array} \right)$$

$\Omega(A)$; 11, 12 2.4.1
 2.4.0 2.4.2, (2.4.0,
), , 2.32 2.4.1,
 (2.4.0) (12). 2.4.3
 2.4.0 2.4.2
 2.1.7.
 2.4.2,
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 " "

Содержание работы

$\mathcal{H}(X)$,
 X
 \mathbb{R} .
 2.0.1. A ,
 1.5.1. $A \ B$
 $A \times B$.
 1.5.2. $R \ A \ B$

$$\pi_A : A \times B \rightarrow A \quad \pi_B : A \times B \rightarrow B$$

R

2.1.1. $K \in \mathcal{H}(X)$. $R(K) \subset K \times \tilde{A}$

$: (p, a_j^i) \in R(K), |pa_j^i| \leq d_i.$

$R(K)$ d -

$A = \{A_1, \dots, A_n\} \subset \mathcal{H}(X)$

$\tilde{A} = \sqcup_i A_i, d = \{d_1, \dots, d_n\} \in \Omega(A) \quad K_d \in \Sigma_d(A)$

A

$A_i \in A \quad m_i,$

$a_j^i.$

$A_i = \bigcup_{j=1}^{m_i} \{a_j^i\}.$

$B_{d_i}(a_j^i), B_{d_i}(A_i), U_{d_i}(a_j^i), U_{d_i}(A_i)$

$B_j^i, B^i, U_j^i \quad U^i$

1

$P \quad Q, \quad Q$

$C_i.$

$p \in P$

$C_i \quad p$

$q \in Q,$

q

$p,$

1

2.2 (. . . , . . . , . . .). [24].

$K \in \mathcal{H}(X).$

$R(K)$

1

K

$\Sigma_d(A).$

$R(K)$

1 (. . .), K

2.1 (. . .).

$R(K)$

1

K

1.12

$p \in K$

$R(p)$

$A_i,$

$p \in B_j^i$

$j,$

$K \subset B^i$

$i, \dots K \subset K_d,$

(1)

2.1

$R(K)$

$$(p, a_j^i) \in R, \dots |pa_j^i| \leq d_i, \dots \quad a_j^i \in \tilde{A} \quad p \in K \quad (2) \quad 2.1$$

$$K \setminus \{p\} \quad R(p) \quad B_j^i \quad R^{-1}(a_j^i) = \{p\}, \quad (3) \quad 2.1$$

$$K \in \mathcal{H}(X) \quad 2.1 ($$

$$K \subset K_d, \quad p \in K$$

$$B^i, \quad |pa_j^i| \leq d_i, \quad j, \quad R(p)$$

$$A_i, \quad K \cap B_j^i$$

$$R^{-1}(a_j^i) \quad i \quad j, \quad R$$

$$A_i, \quad p \in K \quad a_j^i \in$$

$$A_i, \quad K \setminus \{p\} \quad B_j^i, \quad R^{-1}(a_j^i) =$$

$$\{p\}. \quad R \quad 1. \quad \square$$

2.6 (. . . , . . . , . . .). [24].

$$K_\lambda \in \Sigma_d(A)$$

$$\#\tilde{A} - n + 1,$$

$$A_i, \quad m_i > 1, \quad \#\tilde{A} - n.$$

$$R(K_\lambda)$$

2.2

$$K_\lambda$$

$$R(K_\lambda)$$

1

1.13,

□

2.6

1

$$: \#K_2 = 3 = (m_1 + m_2) - n,$$

$$m_1 = 3, m_2 = 2, n = 2.$$

$$\#\tilde{A} - n + 1.$$

3.

$$A = \{A_1, A_2\},$$

$$A_1 = \{a_1^1, a_2^1\}, A_2 = \{a_1^2\} \in$$

$$\mathcal{H}(\mathbb{R}^2). \quad a_1^1 \quad a_2^1$$

$$a_1^2, \quad \dots \quad 2.3.$$

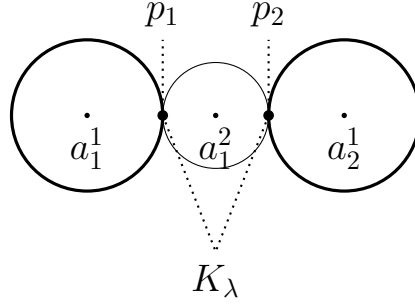
$$\rho = |a_1^2 a_1^1| = |a_1^2 a_2^1|,$$

$$d_H(A_1, A_2) = \rho.$$

$$d_1 \quad d_2$$

$$d_1 > d_2 > 0, d_1 + d_2 = \rho.$$

2.1



2.3

$\#K_\lambda$

$$\{p_1, p_2\} = K_\lambda \in \Sigma_{(d_1, d_2)}(A) \quad (K_\lambda = K_{(d_1, d_2)}).$$

$$\#K_\lambda = 2 = (m_1 + m_2) - n + 1, \quad m_1 = 2, m_2 = 1, n = 2.$$

$$5. \quad 1.13$$

$n \quad m_i.$

$$R \quad 1.13$$

$$1. \quad 2.6 \quad n \quad m_i.$$

$$6. \quad 2.6 \quad \Sigma_d(A)$$

$$K_d = K_\lambda). \quad \Sigma_d(A) \quad (K_d$$

$$4. \quad A = \{A_1, A_2\}, \quad A_1 = \{a_1^1, a_2^1\}, A_2 = \{a_1^2, a_2^2\} \in \mathcal{H}(\mathbb{R}^2).$$

$$2.4. \quad r = |a_1^1 a_1^2| = |a_1^2 a_2^1| = |a_2^1 a_2^2|, \quad d_H(A_1, A_2) = \rho.$$

$$d_1 > d_2 > 0, \quad d_1 + d_2 = \rho. \quad K_{(d_1, d_2)}$$

$$p_1 = B_1^1 \cap B_1^2, p_2 = B_1^2 \cap B_2^1, p_3 = B_2^1 \cap B_2^2.$$

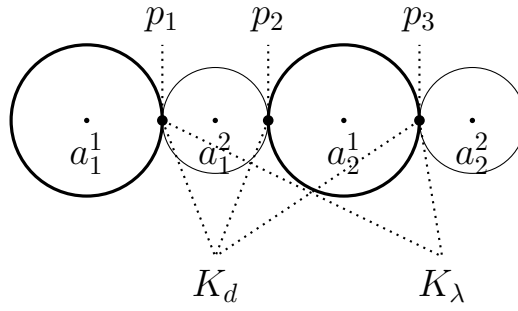
$$G_R \quad R \subset K_{(d_1, d_2)} \times \tilde{A}$$

$$a_1^1 p_1, p_1 a_1^2, a_1^2 p_2, p_2 a_2^1, a_2^1 p_3, p_3 a_2^2, \quad p_2$$

2,

1

$$K = \{p_1, p_3\},$$



$$2.4 \quad K_d \quad K_d \neq K_\lambda$$

2.7 (. . . , . . . , . . .) . [24].

$$X \quad K_d$$

$$\#K_\lambda \leq \#\tilde{A} - n.$$

3

$$m_i \quad 1,$$

$$\#K_d = 1,$$

i

$$m_i > 1.$$

$$R = R(K_\lambda)$$

$$K_\lambda$$

2.2

$$R(K_\lambda)$$

1,

R

2

$$p \in K_\lambda \quad A_i$$

$$R(p) \cap A_i$$

$$K_d = K_d \cap B^i = K_d \cap \left(\bigcup_{j=1}^{m_i} B_j^i \right) = \bigcup_{j=1}^{m_i} (K_d \cap B_j^i).$$

$$K_d$$

2.1.8

$$a_j^i, \quad U_j^i \cap K_d = \emptyset,$$

2.1.10

$$B_j^i \cap K_d$$

$$p \in B_j^i \cap K_d$$

U

$$U \cap (B_j^i \cap K_d) = \{p\}.$$

p

$$K_d \cap B_k^i, \quad k \neq j,$$

$$B_k^i, \quad k \neq j,$$

U

$$U \cap K_d = \{p\}, \quad \dots p$$

$$\begin{aligned}
& B_j^i \cap K_d && K_d && B_k^i, k \neq j. \\
& i \neq j, && && K_\lambda \cap B_j^i \\
& && B_j^i \cap K_d, && p, \\
& && B_k^i, k \neq j. && R(p) \supset \{a_j^i, a_k^i\}, \\
& 1.14 (&&). &&
\end{aligned}$$

□

7.

$$A = \{A_1, \dots, A_n\} \quad A' = \{A_1, \dots, A_n, \{a_1\}, \dots, \{a_k\}\},$$

$$\begin{aligned}
& \{a_1\}, \dots, \{a_k\} && K_\lambda \in \Sigma_d(A) \quad K'_\lambda \in \\
& \Sigma_{d'}(A'). && d \neq d', &&
\end{aligned}$$

2.6 2.7,

$$K_\lambda \quad K'_\lambda,$$

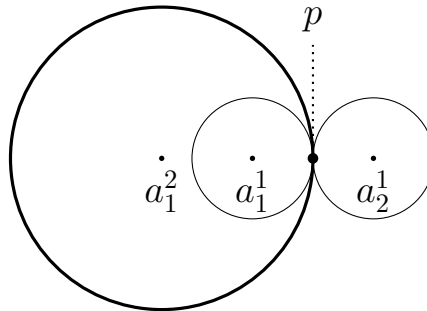
2.6,

2.7

$$5. \quad A = \{A_1, A_2\}, \quad A_1 = \{a_1^1, a_2^1\}, A_2 = \{a_1^2\} \in \mathcal{H}(\mathbb{R}^2).$$

2.5.

$$d_1 < |a_1^2 a_1^1|, \quad d_2 = |a_1^2 a_1^1| + d_1, \quad |a_1^1 a_2^1| = 2d_1, \quad d_H(A_1, A_2) = d_1 + d_2.$$



2.5

#K_\lambda

$$\begin{aligned}
& d = \{d_1, d_2\}, && K_d = B_1^1. && p = B_1^2 \cap B_2^1. \\
& K_\lambda = \{p\}. && K_\lambda \subset K_d. && p \\
& B_2^1, B_1^2 && B_1^1. && K_\lambda \setminus \{p\} = \emptyset, && 2.1, K_\lambda
\end{aligned}$$

$$, m_1 = 2, m_2 = 1, n = 2, \quad \#K_\lambda = 1 = (m_1 + m_2) - n,$$

1.14

$$n \quad m_i.$$

R

2.

2.7

$$n \quad m_i.$$

\tilde{A}

n

$A.$

2.2

$$\#K_\lambda \leq \begin{cases} \# \tilde{A} - n + 1, & ; \\ \# \tilde{A} - n, & A_i, A_j \in A \ (i \neq j) : \\ \#A_i > 1 \quad \#A_j > 1 & \\ & X \\ & K_d \end{cases}$$

$$\{A_1, \dots, A_n\} \subset \mathcal{H}(X)$$

$$d \in \Omega(A).$$

$$K_d^{\text{Conv}} = \bigcap_{i=1}^n B_{d_i}(\text{Conv}(A_i)).$$

$$A = \{A_1, \dots, A_n\} \subset \mathcal{H}(X).$$

$$A^{\text{Conv}} = \{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\}.$$

2.4.1.

$$A = \{A_1, \dots, A_n\} \subset \mathcal{H}(X)$$

$$S_A = S_{A^{\text{Conv}}},$$

A

$$S_A \geq S_{A^{\text{Conv}}}.$$

$$A = \{A_1, \dots, A_n\} ($$

),

$$d \in \Omega(A)$$

$$K_d^{\text{Conv}} = \bigcap_{i=1}^n B_{d_i}(\text{Conv}(A_i)).$$

$$d \in \Omega(A)$$

$$A^{\text{Conv}} = \{\text{Conv}(A_1), \dots, \text{Conv}(A_n)\} :$$

$$- F_d^{\text{Conv}(A_i)} := \{a \in \text{Conv}(A_i) \mid U_{d_i}(a) \cap K_d^{\text{Conv}} = \emptyset\};$$

$$- \text{HP}(F_d^{\text{Conv}(A_i)}) := B_{d_i}(F_d^{\text{Conv}(A_i)}) \cap K_d^{\text{Conv}};$$

$$- \text{HP}(F_d^{A^{\text{Conv}}}) := \bigcup_i \text{HP}(F_d^{\text{Conv}(A_i)});$$

$$- L_d^{\text{Conv}(A_i)} := \{a \in \text{Conv}(A_i) \mid \text{Int}(B_{d_i}(a) \cap K_d^{\text{Conv}}) = \emptyset\}.$$

2.4.2

2.21

11 (. . .)

$$). [26]. \quad A = \{A_1, \dots, A_n\} \quad d \in \Omega(A).$$

$$i \quad F_d^{\text{Conv}(A_i)} = \emptyset \quad i \quad L_d^{\text{Conv}(A_i)} = \emptyset,$$

 A A

2.4.2

$$d \in \Omega(A^{\text{Conv}})$$

 K_d^{Conv}

$$\Sigma_d(A^{\text{Conv}}).$$

$$i \quad F_d^{\text{Conv}(A_i)} = \emptyset.$$

$$d \in \Omega(A^{\text{Conv}})$$

 A^{Conv}

2.21

$$i \quad L_d^{\text{Conv}(A_i)} = \emptyset.$$

 $L_d^{\text{Conv}(A_i)}$

$$a \in \text{Conv}(A_i)$$

$$\emptyset \neq \text{Int}(B_{d_i}(a) \cap K_d^{\text{Conv}}) = \text{Int} B_{d_i}(a) \cap \text{Int} K_d^{\text{Conv}}.$$

$\text{Int } K_d^{\text{Conv}} \neq \emptyset.$

8

A^{Conv}

$$L_d^{\text{Conv}(A_i)} = F_d^{\text{Conv}(A_i)}.$$

$\emptyset,$

$$F_d^{\text{Conv}(A_i)} =$$

2.21

$$A = \{A_1, \dots, A_n\}$$

□

2.4.2

2.28

12 (. . .

). [26].

$$A = \{A_1, \dots, A_n\}$$

d_i

$$d \in \Omega(A).$$

$$\text{HP}(F_d^{\text{Conv}(A_s)}) = \emptyset$$

$$p \in \text{HP}(F_d^{A^{\text{Conv}}})$$

$$p \notin \partial B_{d_s}(\text{Conv}(A_s)),$$

A

2.4.2

A

$$K_d^{\text{Conv}}$$

$$\Sigma_d(A^{\text{Conv}}).$$

$$p \in \text{HP}(F_d^{A^{\text{Conv}}})$$

$$p \notin \text{HP}(F_d^{\text{Conv}(A_s)}) \left(\text{HP}(F_d^{\text{Conv}(A_s)}) = \emptyset \right)$$

$$p \notin \partial B_{d_s}(\text{Conv}(A_s)).$$

A^{Conv}

d_i

2.28

$$A = \{A_1, \dots, A_n\}$$

□

$$U_d^{\text{Conv}} = \text{Int } K_d^{\text{Conv}}.$$

$$U_d^{\text{Conv}} = \bigcap_{i=1}^n \text{Int } B_{d_i}(\text{Conv}(A_i)).$$

$K \in \mathcal{H}(X)$

$r > 0$

$$\text{Int } B_r(K) =$$

$$U_r(K), \quad r = 0 \quad \text{Int } K \neq \emptyset$$

$$\text{Int } B_0(K) = \text{Int } K \neq U_0(K) = \emptyset.$$

$$\Sigma_d(A) \quad \text{HP}(D_d^A) \neq \emptyset. \quad A_i$$

3.

- (1) X ;
- (2) $A = \{A_1, \dots, A_n\}$;
- (3) $U_d^{\text{Conv}} = \text{Int } K_d^{\text{Conv}} \neq \emptyset, \quad d \in \Omega(A)$;
- (4) $d_s > 0$;
- (5) $\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \text{HP}(D_d^A) \subset U_d^{\text{Conv}}$.

$$\text{HP}(D_d^A) = \text{HP}(L_d^A), \quad (5) \quad 3$$

$$\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \text{HP}(L_d^A) \subset U_d^{\text{Conv}}. \quad (2.23)$$

$$\text{Cl}(\text{Int } K_d) = K_d, \quad 2.23$$

$$\text{HP}(D_d^A) = \text{HP}(L_d^A) = \text{HP}(F_d^A), \quad (5)$$

3 (2.23)

$$\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \text{HP}(F_d^A) \subset U_d^{\text{Conv}}. \quad (2.24)$$

2.32

$$(1)-(5) \quad 3 \quad K_d^{\text{Conv}}$$

2.28

$$d_i > 0, \quad 12 \quad K_d^{\text{Conv}} \quad A$$

$$(1)-(5) \quad 3 \quad \delta' > 0, \quad 0 < \delta \leq \delta'$$

$$S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S\left(A^{\text{Conv}}, B_{d_s-\delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}}\right) \geq \delta. \quad (2.29)$$

(2.29)

A.

2.4.1

$$S(A^{\text{Conv}}, K_d^{\text{Conv}}) \leq S_A.$$

$$\begin{aligned} 0 < \delta &\leq S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S\left(A^{\text{Conv}}, B_{d_s-\delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}}\right) \leq \\ &\leq S_A - S\left(A^{\text{Conv}}, B_{d_s-\delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}}\right) \leq S_A - S_{A^{\text{Conv}}}. \end{aligned}$$

A

(1)–(5)

3

A

d_i

$$d \in \Omega(A),$$

12

2.37.

(1)–(5)

3.

$$\emptyset \neq \text{HP}(D_d^A) \subset U_{d_s}(\text{Conv}(A_s)).$$

12.

2.2.5

(1)

(2)

3

$$\text{HP}(D_d^A) = \text{HP}(L_d^A),$$

2.37

3

$$\text{HP}(L_d^A) \subset U_{d_s}(\text{Conv}(A_s)).$$

$$\text{Cl}(\text{Int } K_d) = K_d, \quad 2.23,$$

$$\text{HP}(D_d^A) = \text{HP}(L_d^A) = \text{HP}(F_d^A) \subset U_{d_s}(\text{Conv}(A_s)).$$

$$2.4.2. \quad K_\lambda \in \Sigma_d(A)$$

$$K_\lambda \setminus \text{HP}(D_d^A) \subset \text{Int } K_d.$$

$$2.38 \text{ (. . .)}. [26]. \quad (1)-(5) \quad 3$$

$$K_\lambda \in \Sigma_d(A)$$

$$\delta_1 := \left| K_\lambda \partial B_{d_s}(\text{Conv}(A_s)) \right| > 0.$$

2.37

$$K_\lambda \cap \text{HP}(D_d^A) \subset U_{d_s}(\text{Conv}(A_s)). \quad (2.34)$$

$$K_\lambda \setminus \text{HP}(D_d^A) \subset \text{Int } K_d \subset U_d^{\text{Conv}} \subset U_{d_s}(\text{Conv}(A_s)). \quad (2.35)$$

(2.34) (2.35)

$$K_\lambda \subset U_{d_s}(\text{Conv}(A_s)).$$

$$\left| K_\lambda \partial B_{d_s}(\text{Conv}(A_s)) \right| > 0.$$

□

$$2.39 \text{ (. . .)}. [26]. \quad (1)-(5) \quad 3$$

$$\delta_2 := \left| \text{Conv}(A_s) \partial B_{d_s}(K_d^{\text{Conv}}) \right| > 0.$$

2.33

$$\text{Conv}(A_s) \subset U_{d_s}(K_d^{\text{Conv}}).$$

$$A_s \subset U_{d_s}(K_d^{\text{Conv}}).$$

$$a \in A_s \quad : \quad B_{d_s}(a) \cap K_d$$

$$B_{d_s}(a) \cap K_d$$

$$A_s \subset B_{d_s}(K_d).$$

$$B_{d_s}(a) \cap K_d \quad , \quad a \in D_d^{A_s} \quad B_{d_s}(a) \cap K_d = \text{HP}(a, D_d^{A_s}).$$

$$\partial B_{d_s}(a) \cap K_d \subset \text{HP}(a, D_d^{A_s}). \quad \text{HP}(a, D_d^{A_s}) \subset K_d. \quad \partial B_{d_s}(a) \cap K_d = \partial B_{d_s}(a) \cap \text{HP}(a, D_d^{A_s}). \quad (3) \quad (5) \quad 3$$

$$\partial B_{d_s}(a) \cap K_d \subset U_d^{\text{Conv}}. \quad (2.36)$$

$$\partial B_{d_s}(a) \cap K_d = \emptyset. \quad U_{d_s}(a) \cap K_d \neq \emptyset, \quad B_{d_s}(a) \cap K_d \neq \emptyset. \\ \emptyset. \quad K_d \subset K_d^{\text{Conv}} \quad U_{d_s}(a) \cap K_d^{\text{Conv}} \neq \emptyset. \\ \partial B_{d_s}(a) \cap K_d \neq \emptyset. \quad (2.36) \quad \partial B_{d_s}(a) \cap$$

$$K_d \cap U_d^{\text{Conv}} \neq \emptyset. \quad \partial B_{d_s}(a) \cap U_d^{\text{Conv}} \neq \emptyset.$$

$$U_d^{\text{Conv}} \quad , \quad 1.17 (\quad) \quad U_{d_s}(a) \cap U_d^{\text{Conv}} \neq \emptyset.$$

$$B_{d_s}(a) \cap K_d \quad U_{d_s}(a) \cap K_d^{\text{Conv}} \neq \emptyset.$$

$$: B_{d_s}(a) \cap K_d$$

$$B_{d_s}(a) \cap K_d^{\text{Conv}}$$

$$(1) \quad 3 \quad K_d \subset K_d^{\text{Conv}}. \quad K_d^{\text{Conv}} \quad U_{d_s}(a) \cap K_d^{\text{Conv}} \neq \emptyset.$$

$$a \in A_s \quad U_{d_s}(a) \cap K_d^{\text{Conv}} \neq \emptyset,$$

$$A_s \subset U_{d_s}(K_d^{\text{Conv}}).$$

$$K_d^{\text{Conv}} \quad 1.2.3$$

$$(\quad) \quad U_{d_s}(K_d^{\text{Conv}})$$

$$\text{Conv}(A_s) \subset \text{Conv}(U_{d_s}(K_d^{\text{Conv}})) = U_{d_s}(K_d^{\text{Conv}}). \quad (2.37)$$

$$\text{Conv}(A_s) \quad (1)-(5) \quad 3$$

$$|\text{Conv}(A_s) \cap \partial B_{d_s}(K_d^{\text{Conv}})| > 0.$$

□

2.40 (. . .).

). [26].

(1)-(5) 3 . $A = \{A_1, \dots, A_n\}$.
 2.38 2.39 $\delta_1 >$
 $0 \quad \delta_2 > 0,$, $\min\{\delta_1, \delta_2, d_s\} > 0.$ $0 < \delta \leq$
 $\min\{\delta_1, \delta_2, d_s\}$

$$K = B_{d_s - \delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}}. \quad (2.38)$$

:

$$S_A - S_{A^{\text{Conv}}} \geq S_A - S(A^{\text{Conv}}, K) \geq S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S(A^{\text{Conv}}, K) \geq \delta > 0. \quad (2.39)$$

(2.39)

A.

(2.39).

2.41.

$$d_H(\text{Conv}(A_s), K) \leq d_s - \delta < d_s.$$

(2.37), 2.39, $K_d^{\text{Conv}},$
 d_s 1.10 (. . .) $\text{Conv}(A_s) \subset$
 $B_{d_s - \delta}(K_d^{\text{Conv}}).$, $a \in \text{Conv}(A_s)$

$$B_{d_s - \delta}(a) \cap K_d^{\text{Conv}} \neq \emptyset.$$

$$(2.38) \quad a \in \text{Conv}(A_s)$$

$$B_{d_s - \delta}(a) \cap K = B_{d_s - \delta}(a) \cap B_{d_s - \delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}} = B_{d_s - \delta}(a) \cap K_d^{\text{Conv}} \neq \emptyset.$$

$$\text{Conv}(A_s) \subset B_{d_s - \delta}(K). \quad (2.40)$$

$$(2.38) \quad (2.40) \quad d_H(\text{Conv}(A_s), K) \leq d_s - \delta < d_s.$$

□

$\text{Conv}(A_i), i \neq s.$

2.42.

$$d_H(\text{Conv}(A_i), K) \leq d_i.$$

$$K \subset K_d^{\text{Conv}} \subset B_{d_i}(\text{Conv}(A_i)). \quad (2.41)$$

$$\text{Conv}(A_i) \subset B_{d_i}(K).$$

2.4.4 (.)

$$X \quad A \quad \Sigma_d(A) \\ K_\lambda \in$$

$\Sigma_d(A)$

2.38,

$$\text{Conv}(A_s),$$

d_s

1.10 (.)

)

$$K_\lambda \subset B_{d_s-\delta}(\text{Conv}(A_s)).$$

$$K_\lambda \subset K_d \subset K_d^{\text{Conv}},$$

(2.38)

$$K_\lambda \subset B_{d_s-\delta}(\text{Conv}(A_s)) \cap K_d^{\text{Conv}} = K.$$

$$K_\lambda \in \Sigma_d(A)$$

$$A_i \subset B_{d_i}(K_\lambda) \subset B_{d_i}(K). \quad (2.42)$$

K

$B_{d_i}(K)$

1.2.2

(2.42),

$$\text{Conv}(A_i) \subset B_{d_i}(K).$$

(2.41) $i \neq s$

$$d_H(\text{Conv}(A_i), K) \leq d_i.$$

□

2.43.

$$S_A - S_{A^{\text{Conv}}} \geq S_A - S(A^{\text{Conv}}, K) \geq S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S(A^{\text{Conv}}, K) \geq \delta > 0.$$

2.41 2.42

$$S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S(A^{\text{Conv}}, K) \geq \delta > 0.$$

2.4.1

$$S_A \geq S(A^{\text{Conv}}, K_d^{\text{Conv}}).$$

$$S_A - S_{A^{\text{Conv}}} \geq S_A - S(A^{\text{Conv}}, K) \geq S(A^{\text{Conv}}, K_d^{\text{Conv}}) - S(A^{\text{Conv}}, K) \geq \delta > 0.$$

□

2.43

$$A = \{A_1, \dots, A_n\}$$

□

11,

12

2.40

Заключение

$$A = \{A_1, \dots, A_n\}.$$

(2.2),

(2.1.4

).

2.1.7

(1)

$$\tilde{d} = (\tilde{d}_1, \dots, \tilde{d}_n) \in \mathbb{R}^n$$

$D_{\tilde{d}}^{A_i}$

A_i

$$\begin{aligned}
& \tilde{d}_i, & F_{\tilde{d}}^{A_i}, & A_i, & \tilde{d} \\
& L_{\tilde{d}}^{A_i}, & A_i, & \tilde{d}. & , \\
a \in A_i & , & \#B_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) < \infty, & \\
U_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) = \emptyset, & & \text{Int}(B_{\tilde{d}_i}(a) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j)) = \emptyset, & \\
& \cdot & \text{2.1.2, 2.2.1 2.2.2.} & \\
& \text{HP}(p, Y_{\tilde{d}}^{A_i}), & \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j), & p \in Y_{\tilde{d}}^{A_i}, \\
Y \in \{D, F, L\}, & & B_{\tilde{d}_i}(p) \cap \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j), & \\
\text{HP}(Y_d^{A_i}) = \bigcup_{p \in Y_d^{A_i}} \text{HP}(p, Y_d^{A_i}), & & \bigcap_{j=1}^n B_{\tilde{d}_j}(A_j) & \\
A_i, & & Y \in \{D, F, L\}. & \\
& , & & X \\
& & & d \\
- & & A & A_i, \\
\text{HP}(D_d^{A_i}), & & D_d^{A_i} & 2.4 2.22 \\
& , & & A_i \\
& , & X & \\
- & & D_d^{A_i}. & \\
d & & 2.1.7 & \\
& , & & \\
& & D_d^{A_i} \text{ HP}(D_d^{A_i}) & \\
& , & & \\
& & 2.4.2 & \\
X & & d & \dots \\
& , & \text{HP}(p, D_d^{A_i}) & \\
& & & A_j \\
A_k, & j & k, & i, \\
& & 2.1.12. & \\
& & p \in D_d^{A_i} & K_\lambda \\
\text{HP}(p, D_d^{A_i}) (& & 2.1.11 &), \\
& & X & d_i
\end{aligned}$$

d
 2.10
 $|ap| = d_i$
 d_i
 $a \in A_i$
 $K_\lambda \cap \bigcup_{j=1}^n \text{HP}(D_d^{A_j})$
 $p \in X$
 $6 (\dots)$
 K_λ
 $d_i \quad d_k$
 2.20
 2.1.7
 (\dots)
 X
 $F_d^{A_i}$
 $\text{HP}(F_d^{A_i})$
 2.21
 d
 X
 2.15
 X
 2.4.1
 $2.28 (\dots)$
 K_d
 A_i
 $\partial B_{d_i}(A_i) \cap \bigcup_{j=1}^n \text{HP}(F_d^{A_j})$
 2.28
 X
 2.28
 2.10

$$\bigcup_{j=1}^{n'} \text{HP}(F_d^{A_j}).$$

d_i

K_d

$L_d^{A_i}$

i

(

8

$F_d^{A_i}$

)

$\text{Int } K_d \neq \emptyset,$

d

$A_i,$

$L_d^{A_i}$

$\text{HP}(L_d^{A_i})$

X

i

$D_d^{A_i}$

$L_d^{A_i}$

(

2.2.5

).

d

$A_i,$

$L_d^{A_i}$

$\text{HP}(L_d^{A_i})$

X

$\text{Cl}(\text{Int } K_d) = K_d$

d

A_i

(

2.2.3

).

$F_d^{A_i}$

A_i

$$A \cap B, \quad r \in \left[\min_{a \in A} |aB|, +\infty \right).$$

$$f(r) = B_r(A) \cap B,$$

$$\left(\begin{array}{c} B_r(A) \\ \cap \\ B \end{array} \right), \quad r \in A. \quad 1.15$$

$$\begin{aligned} & - \left(\begin{array}{c} 2.4.1 \quad 2.4.2 \\ \{ \text{Conv}(A_1), \dots, \text{Conv}(A_n) \} \\ A^{\text{Conv}}, \quad \sum_{i=1}^n d_H(A_i, K) \\ S(A, K), \\ S(A, K) \quad A \quad S_A. \\ A = \{A_1, \dots, A_n\} \end{array} \right) : \end{aligned}$$

$$S_A = S_{A^{\text{Conv}}}.$$

$$\begin{aligned} & \Omega(A). \quad 2.4.1 \quad 2.4.2 \\ & \left(\begin{array}{c} 11 \\ \text{Conv}(A_i) \end{array} \right) \quad d \in \Omega(A) \end{aligned}$$

$$\begin{aligned} & \left(\begin{array}{c} 12 \\ A^{\text{Conv}} \\ \text{Conv}(A_i) \\ d \in \Omega(A), \\ A_s, \\ A^{\text{Conv}}. \end{array} \right) \quad d \quad 12, \\ & A, \end{aligned}$$

A_s

:

$$\bigcup_{i=1}^n \text{HP}(F_d^{\text{Conv}(A_i)}) \subset U_{d_s}(\text{Conv}(A_s)).$$

(2.40)

X

d

d_s

A

$$\bigcap_{i=1}^n B_{d_i}(\text{Conv}(A_i)),$$

U_d^{Conv} ,

$$\left(\bigcup_{j=1}^{m_s} \partial B_{d_s}(a_j^s) \right) \cap \bigcup_{i=1}^n \text{HP}(D_d^{A_i}) \subset U_d^{\text{Conv}}.$$

A

2.40.

$F_d^{\text{Conv}(A_i)}$

$\text{Conv}(A_i)$,

11 12.

2.40

$\delta > 0$

K ,

$$S_A - S(A^{\text{Conv}}, K) \geq \delta > 0.$$

2.40

2.4.4

1 ()

2.1.7

2.4.3

2.40

$S_A - S_{A^{\text{Conv}}}$.

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